Ponderomotive force of quasiparticles in a plasma

L. O. Silva,[1] R. Bingham,[2] J. M. Dawson,[1] and W. B. Mori[1]
[1]*Department of Physics and Astronomy, University of California at Los Angeles, Los Angeles, California 90095*
[2]*Rutherford Appleton Laboratory, Chilton, Didcot, Oxon OX11 OQX, United Kingdom*
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We derive the force exerted on the background plasma by an arbitrary distribution of noninteracting quasiparticles, corresponding to either collective excitations of the plasma (plasmons and phonons) or dressed particles (photons and neutrinos). Our approach is based on the effective Hamiltonian describing the quasiclassical dynamics of the individual particles in the presence of a background medium. We recover the usual results for the relativistic ponderomotive force of a photon gas and a plasmon gas and we derive the force, due to weak interactions, exerted by the electron neutrinos in a background medium containing electrons, positrons, and neutrons with arbitrary distribution functions. Generalization to other background species and other neutrino flavors is also discussed. [S1063-651X(99)04102-1]

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I. INTRODUCTION

The ponderomotive force of electromagnetic waves [1–6] is a key concept in plasma physics [7,8] and plays a central role in our present understanding of intense laser-plasma interactions [9]. This force arises whenever a nonuniform oscillating electric field is present in a dielectric and can be seen as a slow time scale effect, or the average effect, due to some nonuniformity of the high-frequency oscillations of the electric field [10].

In general, the derivation of the ponderomotive force is based on the analysis of the single-particle dynamics of charged particles in the presence of the electric field [1–6] or in terms of the Maxwell equations for a macroscopic media [10]. By averaging the motion of the charged particle over the fast time scale, corresponding to the high-frequency oscillations of the field, the slow time scale dynamics of the individual particles can then be calculated and the net effect of the electromagnetic forces acting on the particle can be reduced to the ponderomotive force. In this paper we present a different approach, which allows us not only to rederive the previous results, but also to generalize, in a straightforward way, the concept of ponderomotive force to other physical conditions or other *quasiparticles* besides photons (e.g., *dressed* neutrinos), where the dynamics of a single electron is not easily described in terms of the classical force in the relativistic equations of motion.

The starting point of our treatment is a semiclassical description of the fields interacting with the plasma. The fields are described by their equivalent *quasiparticles*, or elementary quanta: photons for the electromagnetic field, plasmons for the longitudinal electrostatic oscillations, phonons for the ion acoustic oscillations, and dressed neutrinos for the neutrino field interacting with the background medium. This is done by defining a distribution function for the particles from the field intensity. Such a description is very useful because a kinetic equation can be derived for the correct quasiparticles distribution. The kinetics or equations of motion for the *quasiparticles* in the plasma are described by the dispersion relation, or the effective Hamiltonian, for each one of the fields or *quasiparticles*. The effective Hamiltonian can be derived from either a classical or a quantum field theory. The key point of our formalism is that by knowing the effective Hamiltonian describing a single *quasiparticle* dynamics, we are then able to derive the force exerted by a gas of noninteracting elementary quanta in the background plasma: The dispersion relation, or the effective Hamiltonian, generates the equations of motion of the quasiparticles and from conservation of momentum (action and reaction) it generates the force of the quasiparticles in the plasma.

Even if the term *ponderomotive force* was coined to describe the forces acting on a dielectric in an arbitrary nonuniform electric field, in this paper we generalize the concept to the interaction of any nonuniform field with a background medium. In fact, the force exerted in a background medium due to a nonuniform field can also be seen as the pressure gradient arising due to some inhomogeneity in the quasiparticle distribution.

We aim to achieve two objectives: to derive the ponderomotive force solely based on the quasiparticle concept, thus providing an easy tool to generalize and unify the concept for different physical scenarios, and to explicitly show the relation between the ponderomotive force and the wave action density, or the quasiparticle number density. In particular, we will be mainly interested in determining the ponderomotive force of an arbitrary distribution of neutrinos (with different flavors) in an arbitrary background medium.

Our focus will be on the derivation of the force exerted by an arbitrary distribution of *quasiparticles*. In Sec. II we present our formalism and the approximations involved in our description. The general expression for the ponderomotive force is then derived and represented as a function of the effective Hamiltonian or dispersion relation (describing the quasiclassical dynamics of the *quasiparticles*) and the *quasiparticle* distribution function. In Sec. III we apply our results to classical fields arising in a plasma. The ponderomotive force exerted by a gas of photons in a plasma is rederived in the relativistic regime, thus showing the equivalence between our approach and the derivation of the ponderomotive force found in the literature. We also derive the force due to a gas of plasmons in a weakly relativistic plasma and the ponderomotive force of a gas of phonons in a dusty plasma.

In Sec. IV we consider the interaction of a gas of neutrinos with a dense plasma, also containing other species of baryonic matter (such as neutrons, protons, and positrons). The ponderomotive force exerted in the background medium by the neutrinos is then derived. In our derivation we assume that each one of the species in the medium has an arbitrary distribution function, thus generalizing previous results only valid for cold plasmas [11]. Finally, in Sec. V the results of this paper are summarized.

II. GENERAL CONSIDERATIONS

Consider a gas of noninteracting *quasiparticles* (QPs) in a background medium. By *quasiparticles* we mean not only elementary collective excitations of the background medium, such as plasmons and phonons, but also dressed photons or dressed neutrinos. Our formalism is independent of the entities we are considering as QPs, but we assume that the interaction *between* them is negligible. This corresponds to either considering collision frequencies ν_{qq} much smaller than the typical time scale of the process under study (dilute gas) or simply assuming that the interaction between quasiparticles can be discarded. Furthermore, we consider that the dynamics of a single QP is governed by the effective Hamiltonian H_{eff}, which is not only a function of the dynamical variables of the single QP (momentum and position) but is also dependent of the properties of the medium where the QPs propagate.

Using in a direct way the results already known for the ponderomotive force of electromagnetic (e.m.) waves in a dielectric in terms of a gradient in the laser intensity, we could immediately propose an expression for the ponderomotive force, written as a function of the number of photons. This approach would not show, however, the connection between the effective interaction experienced by a single QP and the expression of the ponderomotive force, thus preventing the generalization to other physical conditions. Also, the dependence of the ponderomotive force on the wave action density, or QP number density, would not be clearly stated for nontrivial QP distribution functions. Therefore, a different path must be followed. By determining the free energy of the system, we can relate the changes in the free energy with the work performed by the force exerted by the distribution of particles and from that derive the ponderomotive force.

We can write the total free energy as the sum of two contributions: the free energy of the background medium in the absence of QPs (F_0) and the additional free energy (for a given temperature and density) resulting from the presence of the field elementary excitations. Therefore, the free energy is written as

$$F=F_0(\rho,T)+g_{sq}\int d\mathbf{r}\int\frac{d\mathbf{k}}{(2\pi)^3}f_q(\mathbf{r},\mathbf{k},t)H_{\mathrm{eff}}, \qquad (1)$$

where ρ is the density of the medium, T is the temperature of the medium, f_q describes the QP distribution function in phase space $(\mathbf{r},\mathbf{k})$, $\mathbf{r}$ describes the QP position, and $\mathbf{k}$ is related to the QP momentum $\mathbf{p}_q$ by $\mathbf{k}=\mathbf{p}_q/\hbar$. For the sake of completeness we state here the most important properties of f_q:

$$n_q(\mathbf{r},t)=g_{sq}\int\frac{d\mathbf{k}}{(2\pi)^3}f_q(\mathbf{r},\mathbf{k},t), \qquad (2)$$

$$|\psi(\mathbf{r},t)|^2=g_{sq}\int\frac{d\mathbf{k}}{(2\pi)^3}f_q(\mathbf{r},\mathbf{k},t)H_{\mathrm{eff}}, \qquad (3)$$

$$|\phi(\mathbf{k},t)|^2=g_{sq}\int d\mathbf{r}\,f_q(\mathbf{r},\mathbf{k},t)H_{\mathrm{eff}}, \qquad (4)$$

where n_q is the quasiparticle number density, $|\psi(\mathbf{r},t)|^2$ is the spatial energy density, and $|\phi(\mathbf{k},t)|^2$ is the spectral energy density, so that the second term on the right-hand side of Eq. (1) represents the total free energy due to the presence of QPs. g_{sq} is the QP statistical weight and it accounts for spin degeneracy of the QPs.

We now generalize the procedure described in Ref. [10] to electromagnetic waves. Let us assume that an isothermal deformation $\mathbf{u}$ of the background infinite medium occurs. If the distribution of quasiparticles exerts a force $\mathbf{f}$ over the medium, work δW will be done by that force such that

$$\delta W=\int d\mathbf{r}\,\mathbf{f}\cdot\mathbf{u}, \qquad (5)$$

where $\mathbf{f}$ has the dimensions of force per unit volume. We consider that this force will act only on the electrons of the background medium. The ponderomotive force experienced by the ions can be discarded since it is m_e/m_i smaller than the ponderomotive force experienced by the electrons (m_e is the electron rest mass and m_i is the ion mass). However, inclusion of other background species is straightforward. δW can be related with the change in the free energy for the same displacement $\mathbf{u}$ since $\delta W=-\delta(F-F_0)$. Returning to Eq. (1), we easily obtain

$$\delta F=\delta F_0(\rho,T)+g_{sq}\int d\mathbf{r}\int\frac{d\mathbf{k}}{(2\pi)^3}\times[H_{\mathrm{eff}}\delta f_q(\mathbf{r},\mathbf{k},t)+f_q(\mathbf{r},\mathbf{k},t)\,\delta H_{\mathrm{eff}}]. \qquad (6)$$

We will assume a quasistatic distribution of quasiparticles, which means that a displacement of the background medium will not affect the distribution f_q. Furthermore, f_q is a function only of the dynamical variables. Hence the second term in the integral in Eq. (6) is zero. The effective energy H_{eff} of each QP will be affected by the isothermal background deformation since H_{eff} depends not only on the dynamical variables but also on the properties of the background medium. The change in H_{eff} has two contributions: (i) $\delta_{(1)}H_{\mathrm{eff}}$, due to the fact that particles from the background medium are pushed from $\mathbf{r}-\mathbf{u}$ to $\mathbf{r}$, and (ii) $\delta_{(2)}H_{\mathrm{eff}}$, due to the change of the distribution function, or density, of the background medium in position $\mathbf{r}$. The first contribution for δH_{eff} is

$$\delta_{(1)}H_{\mathrm{eff}}=-\mathbf{u}\cdot\nabla H_{\mathrm{eff}}. \qquad (7)$$

For the remaining contribution, we first use the fact that the relative change in the volume element of the background medium is $dV/V=\nabla\cdot\mathbf{u}$. Therefore, the change in the number density n_{bg} is $\delta n_{\mathrm{bg}}=-n_{\mathrm{bg}}\nabla\cdot\mathbf{u}$, where n_{bg} is the number density of the background particles affected by the ponderomotive force (in our case, just the electrons). In the same way, the change in the distribution function of the background medium particles $f_{bg}(\mathbf{r},\mathbf{p})$ is $\delta f_{bg}(\mathbf{r},\mathbf{p},t) = -f_{bg}(\mathbf{r},\mathbf{p})\nabla\cdot\mathbf{u}$, where $\mathbf{p}$ is the momentum of the background particles (we also assume that the isothermal deformation does not impart momentum to the medium). Therefore, $\delta_{(2)}H_{\rm eff}$ can be written as either

$$\delta_{(2)}H_{\rm eff} = -\left(\frac{\partial H_{\rm eff}}{\partial n_{bg}}\right)_T n_{bg}\nabla\cdot\mathbf{u} \tag{8}$$

if $H_{\rm eff}$ is a function of n_{bg} or

$$\delta_{(2)}H_{\rm eff} = -\left(\frac{\partial H_{\rm eff}}{\partial f_{bg}}\right)_T f_{bg}(\mathbf{r},\mathbf{p})\nabla\cdot\mathbf{u} \tag{9}$$

for a more general dependence of $H_{\rm eff}$ on the distribution function of the background medium f_{bg}. Inserting Eqs. (7) and (8) in Eq. (6), we obtain

$$\delta F - \delta F_0(\rho,T) = -g_{sq}\int d\mathbf{r}\int\frac{d\mathbf{k}}{(2\pi)^3}f_q(\mathbf{r},\mathbf{k},t)\times\left[\mathbf{u}\cdot\nabla H_{\rm eff} + \left(\frac{\partial H_{\rm eff}}{\partial n_{bg}}\right)_T n_{bg}\nabla\cdot\mathbf{u}\right]. \tag{10}$$

Performing an integration by parts over the second term on the right-hand side of Eq. (10) results in

$$\delta F - \delta F_0(\rho,T) = -g_{sq}\int d\mathbf{r}\int\frac{d\mathbf{k}}{(2\pi)^3}\mathbf{u}\cdot f_q(\mathbf{r},\mathbf{k},t)\nabla H_{\rm eff} + g_{sq}\int d\mathbf{r}\int\frac{d\mathbf{k}}{(2\pi)^3}\mathbf{u}\cdot\nabla\times\left[f_q(\mathbf{r},\mathbf{k},t)\left(\frac{\partial H_{\rm eff}}{\partial n_{bg}}\right)_T n_{bg}\right]. \tag{11}$$

Therefore, the ponderomotive force, per unit volume, acting on the medium is

$$\mathbf{f}(\mathbf{r},t) = g_{sq}\int\frac{d\mathbf{k}}{(2\pi)^3}f_q(\mathbf{r},\mathbf{k},t)\nabla H_{\rm eff} - g_{sq}\int\frac{d\mathbf{k}}{(2\pi)^3}\nabla\left[f_q(\mathbf{r},\mathbf{k},t)\left(\frac{\partial H_{\rm eff}}{\partial n_{bg}}\right)_T n_{bg}\right]. \tag{12}$$

A similar expression can also be derived when $H_{\rm eff}$ depends on the distribution function of the background medium f_{bg}. This will be discussed for the particular case of the ponderomotive force due to neutrinos in a plasma. For a linear dependence of $H_{\rm eff}$ on n_{bg}, i.e., $n_{bg}(\partial H_{\rm eff}/\partial n_{bg}) = H_{\rm eff}$, Eq. (12) can be further simplified to

$$\mathbf{f}(\mathbf{r},t) = -g_{sq}\int\frac{d\mathbf{k}}{(2\pi)^3}H_{\rm eff}\nabla f_q(\mathbf{r},\mathbf{k},t). \tag{13}$$

Equation (12), as well as the appropriate definition of $H_{\rm eff}$, is the starting point of our discussion of the ponderomotive force due to different types of QPs propagating in a plasma. It also represents a generalization for arbitrary fields of the Landau-Lifshitz arguments [10].

III. PONDEROMOTIVE FORCE OF PHOTONS, PLASMONS, AND PHONONS

Having determined the general expression for the ponderomotive force of a gas of noninteracting QPs, we now proceed by evaluating Eq. (12) for different classical fields or classical QPs. We first consider the ponderomotive force due to a distribution of photons characterized by $f_q \equiv \mathcal{N}(\mathbf{r},\mathbf{k},t)$. The number of photons $\mathcal{N}$ obeying the properties (2)–(4) can be obtained from the Wigner function of the electromagnetic field [12]. The concept of the number of photons was introduced in the plasma physics literature in the 1960s, associated with the random phase approximation [13]. We point out, however, that a proper definition of the number of photons based on the Wigner function can describe any e.m. field configuration (see the Appendix for a detailed discussion of this problem). For the number of photons $\mathcal{N}$, the role of the effective Hamiltonian $H_{\rm eff}$ is played by $\hbar\omega(\mathbf{r},\mathbf{k},t) \equiv \hbar\omega_{\rm k}$, where the frequency $\omega_{\rm k}$ is obtained from the dispersion relation for the electromagnetic waves propagating in the plasma. This is equivalent to assuming that each single photon obeys the dispersion relation for plane electromagnetic waves. Since we are considering an arbitrary distribution of photons, the relativistic mass correction must also be included in the dispersion relation for circularly polarized electromagnetic plane waves:

$$\omega_{\rm k}^2 = k^2c^2 + \frac{\omega_{pe}^2(\mathbf{r},t)}{\gamma}, \tag{14}$$

where $\omega_{pe}^2(\mathbf{r},t) = 4\pi e^2 n_e(\mathbf{r},t)/m_e$ is the local electron plasma frequency and $n_e(\mathbf{r},t)$ is the electron number density. γ is the relativistic mass correction factor, which is a function of the electric field intensity and the energy distribution function of the electrons in the background plasma. For the sake of clarity, we will not write down the explicit expression for γ since it will not add any additional features to our derivation. We note that in Eq. (14), $\mathbf{k}$, $\mathbf{r}$, and t are independent variables. Furthermore, Eq. (14) is valid in the limits of the geometrical optics approximation [14], i.e.,

$$\frac{\omega_{\mathbf{k}}}{2\pi} \gg \left|\frac{\partial}{\partial t}\ln n_e(\mathbf{r},t)\right|, \quad \frac{|\mathbf{k}|}{2\pi} \gg |\nabla\ln n_e(\mathbf{r},t)|. \tag{15}$$

These limits establish the conditions under which this approach is valid: Whenever two very different time scales are present, we can treat the high-frequency perturbations as a gas of quasiparticles propagating in a background with slow density modulations.

Using Eq. (14) in Eq. (12) and the fact that

$$\left(\frac{\partial H_{\rm eff}}{\partial n_{bg}}\right)_T n_{bg} \equiv \left(\frac{\partial\hbar\omega_{\mathbf{k}}}{\partial\omega_{pe}^2}\right)_T\omega_{pe}^2 = \frac{\hbar\omega_{pe}^2}{2\omega_{\mathbf{k}}\gamma} \tag{16}$$

and

$$\nabla H_{\rm eff} \equiv \nabla \hbar \omega_{\mathbf{k}} = \frac{\hbar}{2\omega_{\mathbf{k}}} \nabla \frac{\omega_{pe}^2}{\gamma}, \tag{17}$$

we obtain, after some algebra, the ponderomotive force due to a photon distribution $\mathcal{N}$:

$$\mathbf{f}_{\rm ph}(\mathbf{r},t) = -\frac{\omega_{pe}^2}{2\gamma} g_{\rm ph} \nabla \int \frac{d\mathbf{k}}{(2\pi)^3} \hbar \frac{\mathcal{N}}{\omega_{\mathbf{k}}}, \tag{18}$$

where $g_{\rm ph}=1$ (2) for circular (linear) polarization. For plane electromagnetic waves, $\mathcal{N}=(|E_0|^2/8g_{\rm ph}\pi\hbar\omega_0)\,\delta(\mathbf{k}-\mathbf{k}_0)$, with E_0 the electric field amplitude and $\omega_0(k_0)$ the frequency (wave number) of the electromagnetic field [12]. In this case, the ponderomotive force acting on a single electron reduces to the more familiar form

$$\mathbf{f}_{\rm ph}(\mathbf{r},t) = -\frac{e^2}{2m_e\gamma} \nabla \mathbf{A}^2, \tag{19}$$

where $\mathbf{A}$ is the high-frequency vector potential. It is straightforward to see that ponderomotive force effects can arise due to two different conditions: inhomogeneity in the number of photons distribution function $\mathcal{N}$ and/or spatial-dependent frequency. The present understanding of ultraintense short laser pulse propagation in plasmas is based on the different roles played by these two contributions [15]. Equation (19) agrees with previous derivations of the relativistic ponderomotive force for circularly polarized photons [1,2,4–6]. For linearly polarized plane electromagnetic waves, the dispersion relation (14) is no longer valid; a different expression for the relativistic ponderomotive force appears [3], which under certain conditions reduces to Eq. (19). As far as we know, Eq. (18) is the first derivation of the relativistic ponderomotive force, using solely the number of photons concept. Also, Eq. (18) is valid for any e.m. field configuration, as long as $\mathcal{N}$ is defined using the proper definition of Wigner function for an arbitrary e.m. field (see the Appendix).

For a gas of plasmons (electron plasma oscillations), an expression similar to Eq. (18) can also be derived in a similar manner, but now the dispersion relation is

$$\omega_{\mathbf{k}}^2 = \omega_{pe}^2 \left(1 - \frac{5}{2}\frac{k_BT_e}{m_ec^2}\right) + 3k^2 \frac{k_BT_e}{m_e}, \tag{20}$$

where T_e is the plasma electron temperature and we have included the first relativistic correction to the dispersion relation, valid under the conditions $\omega_{\mathbf{k}}/k \gg \sqrt{k_BT_e/m_e}$ and $k_BT_e \ll m_ec^2$ [16]. The ponderomotive force of the gas of plasmons, described by $\mathcal{N}_{\rm plasmon}$, acting on the plasma electrons is

$$\mathbf{f}_{\rm pl}(\mathbf{r},t) = -\frac{\omega_{pe}^2}{2}\left(1 - \frac{5}{2}\frac{k_BT_e}{m_ec^2}\right) \nabla \int \frac{d\mathbf{k}}{(2\pi)^3} \hbar \frac{\mathcal{N}_{\rm plasmon}}{\omega_{\mathbf{k}}}. \tag{21}$$

The ponderomotive force of the gas of plasmons in the weakly relativistic regime can then describe the coupling of the electrostatic oscillations with the low-frequency, long-wavelength ion acoustic oscillations [13].

We now consider the ponderomotive force due to a gas of elementary collective ion excitations. Due to the limits of our formalism [see Eq. (15)], this gas of ion oscillations must interact with even lower-frequency and longer-wavelength plasma perturbations. In an unmagnetized electron-ion plasma, such a physical condition cannot be verified. However, dusty plasmas support oscillations with characteristic frequencies (wavelengths) much lower (longer) than those of ion oscillations (ion acoustic waves or ion plasma waves) [17]. The picture of a gas of ion oscillations in a dust-acoustic oscillation is then reasonable. The general linear dispersion relation for ion oscillations is [8]

$$\omega_{\mathbf{k}}^2 = k^2 \frac{\gamma_i k_B T_i}{m_i} + k^2 \frac{\gamma_e k_B T_e}{m_i} \frac{1}{1+\gamma_e k^2 \lambda_e^2}. \tag{22}$$

with T_i the ion temperature, $\gamma_e\,(\gamma_i)$ the electron (ion) adiabatic index, and $\lambda_e=(k_BT_e/4\pi n_e e^2)^{1/2}$ the electron Debye length. Denoting the number density of QP excitations corresponding to ion collective motions by $\mathcal{N}_{\rm ion\ qp}$, we obtain from Eq. (12), using Eq. (22), the force

$$\mathbf{f}_{\rm ion\ qp}(\mathbf{r},t) = -\frac{m_i}{k_BT_e}\frac{\lambda_e^2}{2} \nabla \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\hbar \mathcal{N}_{\rm ion\ qp}}{\omega_{\mathbf{k}}} \times \left(\frac{k^2\gamma_e k_BT_e}{m_i}\frac{1}{1+\lambda_e^2k^2\gamma_e}\right)^2. \tag{23}$$

In the limit of $\gamma_iT_i/m_i \ll \gamma_eT_e/m_i$, the dispersion relation (22) can be approximated by $\omega_{\mathbf{k}}^2 \simeq k^2\gamma_e k_BT_e/m_i(1+\lambda_e^2k^2\gamma_e)$ and Eq. (23) reduces to

$$\mathbf{f}_{\rm ion\ qp}(\mathbf{r},t) = -\frac{m_i}{k_BT_e}\frac{\lambda_e^2}{2} \nabla \int \frac{d\mathbf{k}}{(2\pi)^3} \hbar \mathcal{N}_{\rm ion\ qp}\omega_{\mathbf{k}}^3. \tag{24}$$

Two opposite physical scenarios can now be explored. In the limit of $k\lambda_e \ll 1$, corresponding to ion acoustic oscillations, the dispersion relation is $\omega_{\mathbf{k}} \simeq kc_s$, with the sound speed $c_s=\sqrt{\gamma_e k_BT_e/m_i}$. Thus the ponderomotive force due to a distribution of ion acoustic waves, or a gas of ion acoustic phonons, is

$$\mathbf{f}_{\rm IAP}(\mathbf{r},t) = -\frac{1}{2}\frac{c_s^3}{\omega_{pi}^2} \nabla \int \frac{d\mathbf{k}}{(2\pi)^3} \hbar k^3 \mathcal{N}_{\rm IAP}, \tag{25}$$

with the ion plasma frequency $\omega_{pi}=\omega_{pe}\sqrt{m_e/m_i}$. On the other hand, in the limit of $k\lambda_e \gg 1$, the dispersion relation (22) describes ion plasmas waves such that $\omega_{\mathbf{k}}^2 \simeq \omega_{pi}^2$. In this case, the ponderomotive force verifies

$$\mathbf{f}_{\rm IPW}(\mathbf{r},t) = \omega_{pi} \nabla \int \frac{d\mathbf{k}}{(2\pi)^3} \hbar \mathcal{N}_{\rm IPW} - \frac{3}{2} \nabla \int \frac{d\mathbf{k}}{(2\pi)^3} \hbar \omega_{\mathbf{k}} \mathcal{N}_{\rm IPW}. \tag{26}$$

We then see that the ponderomotive force associated with ion collective excitations [Eqs. (25) and (26)] is significantly different from that associated with photons [Eq. (18)] or plasmons [Eq. (21)]. This derivation must now be verified using the standard plasma physics methods. The coupling between the ion collective motions and the background medium can play a significant role in the (de)stabilization of dust-acoustic oscillations in a dusty plasma. Both these features will be explored in a future work.

IV. PONDEROMOTIVE FORCE OF NEUTRINOS IN DENSE PLASMAS

We now turn to the central result of this paper: the ponderomotive force of neutrinos in a plasma. This force can provide the coupling mechanism responsible for the anomalous scattering of neutrinos and the consequent deposition in the plasma of some part of the neutrino energy released in a supernova explosion [11,18]. This energy deposition plays a key role in the present understanding of supernovae explosions [19]. An intuitive picture of the ponderomotive force is immediate: If in a given region the energy density in the neutrinos is higher than in other regions, a force will be exerted in the background medium by the neutrinos, towards the regions of lower neutrino energy density. This corresponds to the physical picture of the neutrinos trying to push their way into regions of lower energy density.

In order to derive the ponderomotive force due to the electron neutrinos (ν_e) propagating in a background medium we first write down the effective Hamiltonian for a single electron neutrino in an unmagnetized background of electrons characterized by the electron distribution function $f_e(\mathbf{r},\mathbf{p}_e,t)$ [20]:

$$H_{\rm eff}=\sqrt{p_\nu^2c^2+m_\nu^2c^4}+g_{se}\int\frac{d\mathbf{p}_e}{(2\pi)^3}V_{\rm eff}(\mathbf{r},\mathbf{p}_e,t), \quad (27)$$

with $V_{\rm eff}$ given by

$$V_{\rm eff}(\mathbf{r},\mathbf{p}_e,t)=g_V\sqrt{2}G_F f_e(\mathbf{r},\mathbf{p}_e,t)\left(1-\frac{\mathbf{p}_e\cdot\hat{\mathbf{k}}_\nu}{E_e}\right), \quad (28)$$

where G_F is the Fermi constant of weak interaction, $g_V=1/2+2\sin^2\theta_W\simeq1$ is the effective vector coupling constant in the standard model, θ_W is the Weinberg mixing angle, $\mathbf{p}_\nu=\hbar\mathbf{k}$ is the electron neutrino momentum, $\hat{\mathbf{k}}_\nu=\mathbf{p}_\nu/|\mathbf{p}_\nu|$, E_e is the electron energy (a function of the electron momentum $\mathbf{p}_e$), and m_ν is the neutrino mass, which can be set to zero for massless neutrinos. $g_{se}=2$ is the statistical weight for the electrons, corresponding to spin 1/2, and appears because of spin degeneracy. Also, $f_e(\mathbf{r},\mathbf{p}_e,t)$ satisfies

$$n_e(\mathbf{r},t)=g_{se}\int\frac{d\mathbf{p}_e}{(2\pi)^3}f_e(\mathbf{r},\mathbf{p}_e,t). \quad (29)$$

The effective potential in Eq. (28) has been derived using the methods of finite-temperature quantum field theory [21]. Hence our approach here is clearly a semiclassical one: We assume that the interaction of the neutrinos with the electrons is governed by quantum processes (included in $V_{\rm eff}$) and we take into account the Fermi statistics of the phase space density of the particle numbers, but the neutrino dynamics is determined by the classical Hamiltonian obtained using the equivalence principle, and we neglect the spins. This approximation is valid as long as changes in $V_{\rm eff}$ occur over length scales much longer than the neutrino de Broglie wavelength $\lambda_\nu=2\pi/|\mathbf{k}|$ and no spin waves are considered.

We first analyze the contribution of the term $\mathbf{p}_e\cdot\hat{\mathbf{k}}_\nu/E_e$ to the effective Hamiltonian (27). If the neutrinos propagate along a precise direction and assuming an isotropic distribution for the electrons, the integral

$$\int\frac{d\mathbf{p}_e}{(2\pi)^3}f_e(\mathbf{r},\mathbf{p}_e,t)\frac{\mathbf{p}_e\cdot\hat{\mathbf{k}}_\nu}{E_e} \quad (30)$$

averages to zero. Furthermore, if the neutrino distribution is isotropic, when integrating over the contribution of all the neutrinos for the ponderomotive force, a zero average is obtained once again. Therefore, this term only gives a contribution for both anisotropic neutrino *and* electron distribution functions.

Since Eq. (27) is also a function of the distribution function of the electrons in the background medium, Eq. (9) must now be employed to obtain the expression for the ponderomotive force due to the neutrinos:

$$\mathbf{f}_\nu(\mathbf{r},t)=g_{se}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\mathbf{p}_e}{(2\pi)^3}f_\nu(\mathbf{r},\mathbf{k},t)\nabla V_{\rm eff} -g_{se}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\mathbf{p}_e}{(2\pi)^3} \times\nabla\left[f_\nu(\mathbf{r},\mathbf{k},t)\left(\frac{\partial V_{\rm eff}}{\partial f_e}\right)_T f_e(\mathbf{r},\mathbf{p}_e,t)\right], \quad (31)$$

where $f_\nu(\mathbf{r},\mathbf{k},t)$ is the neutrino distribution function, which also verifies the normalization condition

$$n_\nu(\mathbf{r},t)=g_{s\nu}\int\frac{d\mathbf{k}}{(2\pi)^3}f_\nu(\mathbf{r},\mathbf{k},t), \quad (32)$$

where $n_\nu(\mathbf{r},t)$ is the neutrino number density and $g_{s\nu}=1$ is the neutrino statistical weight [neutrinos are completely polarized (left) particles]. Since the effective potential has a linear dependence on $f_e(\mathbf{r},\mathbf{p}_e,t)$, then

$$\left(\frac{\partial V_{\rm eff}}{\partial f_e}\right)_T f_e(\mathbf{r},\mathbf{p}_e,t)=V_{\rm eff} \quad (33)$$

and Eq. (31) reduces to

$$\mathbf{f}_\nu(\mathbf{r},t)=-g_{se}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\mathbf{p}_e}{(2\pi)^3}V_{\rm eff}\nabla f_\nu(\mathbf{r},\mathbf{k},t). \quad (34)$$

Inserting Eq. (28) in Eq. (34), we obtain

$$\mathbf{f}_\nu(\mathbf{r},t)=-\frac{\sqrt{2}}{2}(1+4\sin^2\theta_W)G_F g_{se} \times\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\mathbf{p}_e}{(2\pi)^3}f_e(\mathbf{r},\mathbf{p}_e,t)\nabla f_\nu(\mathbf{r},\mathbf{k},t) +\frac{\sqrt{2}}{2}(1+4\sin^2\theta_W)G_F g_{se}$$

$$\times \int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{d\mathbf{p}_e}{(2\pi)^3} f_e(\mathbf{r},\mathbf{p}_e,t) \frac{\mathbf{p}_e \cdot \hat{\mathbf{k}}_\nu}{E_e} \nabla f_\nu(\mathbf{r},\mathbf{k},t). \quad (35)$$

Using Eqs. (29) and (32), the first term in Eq. (35) can be rewritten as

$$\mathbf{f}_\nu(\mathbf{r},t) = -\frac{\sqrt{2}}{2}(1+4\sin^2\theta_W) G_F n_e(\mathbf{r},t) \nabla n_\nu(\mathbf{r},t). \quad (36)$$

Equation (35) represents the force per unit volume exerted by the neutrinos over the electrons contained in the unit volume. The second term, which accounts for the contributions from anisotropies, is different from zero only when the electron distribution function *and* the neutrino distribution function are anisotropic. This means that even for a beamed neutrino distribution, the second term vanishes for an isotropic plasma. From now on we will consider an isotropic electron distribution function and hence we discard the contribution of the second term in Eq. (35). The first term in Eq. (35) [or Eq. (36)] is then responsible for the strong coupling between the neutrinos and the electrons, as suggested before [18]. From Eq. (36) we can easily derive the force acting on a single electron due to the presence of the neutrino distribution:

$$\mathbf{f}_{\nu-e} = -\frac{\sqrt{2}}{2}(1+4\sin^2\theta_W) G_F \nabla n_\nu(\mathbf{r},t). \quad (37)$$

We stress that Eq. (36) is valid for any neutrino or electron distribution functions.

The generalization of Eqs. (35)–(37) for the force exerted by neutrinos and antineutrinos over electrons, positrons, or neutrons is also straightforward. Changing the effective potential V_{eff} [22] in order to include the weak interaction of the electron neutrinos and antielectron neutrinos with electrons (positrons), we obtain the ponderomotive force over a single electron (positron)

$$\mathbf{f}_{\nu\bar{\nu}-e^+(e^-)} = \mp \frac{\sqrt{2}}{2}(1+4\sin^2\theta_W) \times G_F \nabla [n_{\nu e}(\mathbf{r},t) - n_{\bar{\nu e}}(\mathbf{r},t)], \quad (38)$$

where the minus (plus) sign refers to electrons (positrons), $n_{\bar{\nu e}}(\mathbf{r},t)$ being the antielectron-neutrino number density. The ponderomotive force over a single neutron can also be written as

$$\mathbf{f}_{\nu\bar{\nu}-n} = \frac{\sqrt{2}}{2} G_F \nabla [n_{\nu e}(\mathbf{r},t) - n_{\bar{\nu e}}(\mathbf{r},t)]. \quad (39)$$

As before, we are assuming an unmagnetized background medium and we are discarding the contribution of the anisotropies of the electron/positron/neutron and neutrino/antineutrino distribution functions. Generalization of the ponderomotive force due to other neutrino flavors (τ neutrinos or muon neutrinos) is straightforward as long as the proper effective potentials V_{eff} are considered [22].

It is now important to compare our expression of the ponderomotive force with that introduced by Bingham *et al.* [11] in a phenomenological way or the one derived by Hardy and Melrose [23] using the methods of quantum plasmadynamics [24]. The expression derived by Bingham *et al.* is based on the analogy between the ponderomotive force due to electromagnetic waves (as derived in [10]) and the ponderomotive force due to the neutrinos and it is only valid in the limits of validity of the Landau-Lifshitz expression, i.e., as long as the neutrino flux is assumed to be monochromatic and the energy (or frequency) of the neutrinos is assumed to be constant. In this particular physical scenario Eq. (11) in Ref. [11] is equivalent to our Eq. (36). A comparison with the results derived in Ref. [23] shows that their results are equivalent to those presented here, in the limit of constant background electron/positron number density. The discrepancy is present whenever gradients of the background number density are assumed and it arises from a misinterpretation of the ponderomotive force concept in Ref. [23]. A correct interpretation of the results obtained by the quantum plasmadynamics formalism gives the same expression for the ponderomotive force of electron neutrinos in a background of electrons and positrons as the one derived here [25].

It must be stressed that a long-range interaction force between neutrinos and electrons was identified before in the context of a quantum kinetic treatment of neutrinos in a lepton plasma. An expression equivalent to Eq. (36) is evident in Eq. (18′) of Ref. [26]. The long-range interaction force is associated with an effective charge the neutrinos acquire in a background of electrons [26,27]. Our results allow us to clearly identify the long-range force in [26] as the ponderomotive force due to the weak interaction of the neutrinos with a plasma.

V. SUMMARY

In this paper we have presented a general derivation of the ponderomotive force due to quasiparticles propagating in a background medium. In particular, the results for the relativistic ponderomotive force of photons were recovered. We have also derived the ponderomotive force of a gas of plasmons in a weakly relativistic plasma. The force of a gas of ion collective motions in the background plasma was also presented. We then applied the same techniques to a gas of neutrinos interacting with a background medium through the weak interaction force. The ponderomotive force due to an arbitrary distribution of neutrinos and antineutrinos interacting with either electrons, positrons, or neutrons was derived.

The ponderomotive force derived here is the force on a Lagrangian fluid element or on a single particle from the background medium. It then provides the proper way to build a single-particle or a self-consistent kinetic theory for the interaction of QPs (neutrinos, photons, plasmons, and phonons) with a background medium, which can be applied as the foundation for the study of QPs driven instabilities in a plasma.

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APPENDIX

In order to clarify the meaning and scope of the number of photons concept, we present a calculation of the number of photons $\mathcal{N}$ for several e.m. field configurations. This will allow us to connect this concept with previous definitions found in the literature and to clearly point out that no random phase approximation is assumed when a proper definition for the number of photons is employed.

The number of photons $\mathcal{N}$ obeys the general properties described by Eqs. (3) and (4) for the distribution function f_q. The class of phase-space distribution functions that verify these properties for a given wave field are usually denoted as Wigner functions [28]. The most common representation of the Wigner function for the electric field $\mathbf{E}(\mathbf{r},t)$ is

$$\mathcal{N}(\mathbf{k},\mathbf{r},t)=\frac{1}{8g_{\rm ph}\pi\hbar\,\omega_{\mathbf{k}}(\mathbf{r},t)}\times\int d\mathbf{s}\,\mathbf{E}(\mathbf{r}-\mathbf{s}/2,t)\cdot\mathbf{E}^*(\mathbf{r}+\mathbf{s}/2,t)\exp(i\mathbf{k}\cdot\mathbf{s}), \tag{A1}$$

where $\omega_{\mathbf{k}}(\mathbf{r},t)$ is obtained from the dispersion relation $D(\omega,\mathbf{k},\mathbf{r},t)\equiv 0$. For the wave field described by $\mathcal{N}$, Eq. (2) defines the wave action density and Eq. (3) defines the field energy density. The Wigner function exactly satisfies a kinetic equation that reduces to a Vlasov equation in the short-wavelength high-frequency approximation.

For simplicity, we will consider propagation in vacuum, meaning that $\omega_{\mathbf{k}}=|\mathbf{k}|c$. We now calculate $\mathcal{N}$ for different electric fields. We first start with a monochromatic plane wave, $\mathbf{E}(\mathbf{r},t)=\mathbf{E}_0\exp i(\mathbf{k}_0\cdot\mathbf{r}-\omega_0 t)$, ω_0 being the frequency of the electric field and $\mathbf{k}_0$ the wave vector. The number of photons is then simply written as

$$\mathcal{N}_{\rm plane}(\mathbf{k},\mathbf{r},t)=\frac{|\mathbf{E}_0|^2}{8g_{\rm ph}\pi\hbar|\mathbf{k}|c}\,\delta(\mathbf{k}-\mathbf{k}_0), \tag{A2}$$

describing a monochromatic beam of photons, as expected. This is also the usual result present in the plasma physics literature since the 1960s [29]. A test of the ability of Eq. (A1) for the definition of the number of photons $\mathcal{N}$ to describe the complete structure of the electric field arises whenever the electric field depicts an interference pattern. The most simple case corresponds to an electric field described by the superposition of two plane waves $\mathbf{E}(\mathbf{r},t)=\mathbf{E}_0\exp i(\mathbf{k}_0\cdot\mathbf{r}-\omega_0 t)+\mathbf{E}_1\exp i(\mathbf{k}_1\cdot\mathbf{r}-\omega_1 t)$. The number of photons for this electric field defined by Eq. (A1) is

$$\mathcal{N}_{\rm BW}(\mathbf{k},\mathbf{r},t)=\frac{1}{8g_{\rm ph}\pi\hbar|\mathbf{k}|c}\Bigg[|\mathbf{E}_0|^2\delta(\mathbf{k}-\mathbf{k}_0)+|\mathbf{E}_1|^2\delta(\mathbf{k}-\mathbf{k}_1)+\mathbf{E}_0\cdot\mathbf{E}_1^*\cos(\boldsymbol{\Delta}_k\cdot\mathbf{r}-\Delta_\omega t)\,\delta\left(\mathbf{k}-\frac{\mathbf{k}_0+\mathbf{k}_1}{2}\right)\Bigg], \tag{A3}$$

where $\boldsymbol{\Delta}_k=\mathbf{k}_1-\mathbf{k}_0$ and $\Delta_\omega=\omega_1-\omega_0$. The presence of two photon beams associated with the two plane waves is obvious. Furthermore, a third beam is present, which results from the interference between the two plane waves, showing the characteristic slow modulation of the beat pattern. Using Eq. (3) we can immediately recover the usual energy density for two interfering monochromatic planes waves. It is then clear that the number of photons, as described by Eq. (A1), already contains the information about any interference pattern present in the electric field.

To clarify this point and make a bridge to the usual definitions, we consider the superposition of several monochromatic plane waves, also containing an additional random phase factor $\psi_{\mathbf{k}}$. The electric field is then written as $\mathbf{E}(\mathbf{r},t)=\sum_{\mathbf{k}'}\mathbf{A}_{k'}\exp i(\mathbf{k}'\cdot\mathbf{r}-\omega_{\mathbf{k}'}t+\psi_{\mathbf{k}'})$. For this electric field Eq. (A1) verifies

$$\mathcal{N}_{\rm RP}(\mathbf{k},\mathbf{r},t)=\frac{1}{8g_{\rm ph}\pi\hbar|\mathbf{k}|c}\sum_{\mathbf{k}'}\sum_{\mathbf{k}''}\mathbf{A}_{\mathbf{k}'}\mathbf{A}_{\mathbf{k}''}^*\times\exp i(\boldsymbol{\Delta}_{\mathbf{k}'\mathbf{k}''}\cdot\mathbf{r}-\Delta_{\omega'\omega''}t+\Delta_{\psi'\psi''})\times\delta\left(\mathbf{k}-\frac{\mathbf{k}'+\mathbf{k}''}{2}\right), \tag{A4}$$

where $\boldsymbol{\Delta}_{\mathbf{k}'\mathbf{k}''}=\mathbf{k}'-\mathbf{k}''$, $\Delta_{\omega'\omega''}=\omega'-\omega''$, and $\Delta_{\psi'\psi''}=\psi_{\mathbf{k}'}-\psi_{\mathbf{k}''}$. Once again, the beat interference pattern observed in Eq. (A3) is also present for the terms verifying $\boldsymbol{\Delta}_{\mathbf{k}'\mathbf{k}''}\neq\mathbf{0}$ and $\Delta_{\omega'\omega''}\neq 0$. Equation (A4) is the generalization of Eq. (A3) for the superposition of an arbitrary number of plane waves. We stress that so far no assumptions have been made regarding the properties of the phases $\psi_{\mathbf{k}}$. When the phases $\psi_{\mathbf{k}}$ are random, a phase averaging of Eq. (A4) can be performed. This averaging corresponds to the well known random phase approximation (RPA) [29]. Since the phase averaging is defined as the average of a statistical ensemble of systems differing from one another only in the phase $\psi_{\mathbf{k}}$ or $\Delta_{\psi'\psi''}$, it is obvious that $\langle\exp i\Delta_{\psi'\psi''}\rangle=\delta(\mathbf{k}'-\mathbf{k}'')$, thus leading to the RPA's number of photons

$$\begin{aligned}\mathcal{N}_{\rm RPA}(\mathbf{k},\mathbf{r},t)&=\langle\mathcal{N}_{\rm RP}(\mathbf{k},\mathbf{r},t)\rangle\\&=\frac{1}{8g_{\rm ph}\pi\hbar|\mathbf{k}|c}\sum_{\mathbf{k}'}\sum_{\mathbf{k}''}\mathbf{A}_{\mathbf{k}'}\mathbf{A}_{\mathbf{k}''}^*\times\exp[i(\boldsymbol{\Delta}_{\mathbf{k}'\mathbf{k}''}\cdot\mathbf{r}-\Delta_{\omega'\omega''}t)]\,\delta(\mathbf{k}'-\mathbf{k}'')\times\delta\left(\mathbf{k}-\frac{\mathbf{k}'+\mathbf{k}''}{2}\right)\\&=\frac{1}{8g_{\rm ph}\pi\hbar|\mathbf{k}|c}\sum_{\mathbf{k}'}|A_{\mathbf{k}'}|^2\delta(\mathbf{k}-\mathbf{k}').\end{aligned} \tag{A5}$$

This is the conventional definition of the number of photons, valid only under the limits of the random phase approximation. No interference pattern is present, thus describing independent and noninterfering photon beams.

From this discussion it becomes evident that a definition of the number of photons based on the Wigner function can rigorously describe different e.m. field configurations, including those where interference between different field components is important. In the limit of the random phase approximation, the usual definitions are recovered. It may be argued that the Wigner function presents some pathologies (it is not a positive-definite function), which, at first sight, could prevent its use. However, we stress that the quantities with straightforward physical meaning are the marginals of the Wigner function [Eqs. (2)–(4)] and these possess the correct physical properties.

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